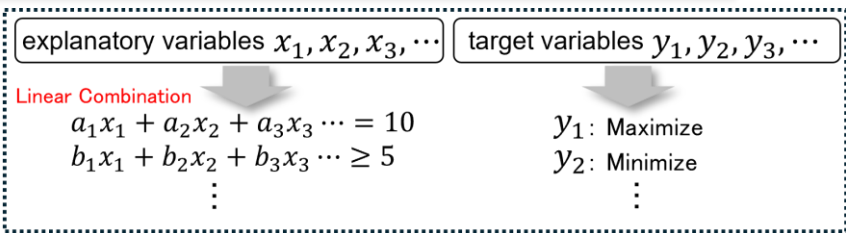


Constrained Optimization Using Multi-Sigma®

1. Constrained Optimization Using Multi-Sigma®

In Multi-Sigma®, multiple constraints can be applied to linear combinations of explanatory variables during optimization. Adding such constraints enables optimization under various operational requirements.



2. Constraints on Total-Sums

The most common linear-sum constraint in optimization is a total constraint. For a mixture of substances A, B, and C, the proportions must sum to 100%. In such cases, imposing the constraint,  $x_1 + x_2 + x_3 = 100$  allows optimization to be performed while satisfying the total-sum requirement.

Similarly, constraints such as requiring the total of substances A and B to be at least 45%, or the total of substances A and C to be at most 30%, can be applied in the same manner. In addition, if optimization is to be performed while maintaining the perimeter of a rectangular design (i.e.,  $2 \times [\text{width} + \text{height}]$ ) at no less than 50 mm, imposing  $2x_1 + 2x_2 \geq 50$  allows optimization while respecting this geometric constraint.

A [%] $x_1$	B [%] $x_2$	C [%] $x_3$	T [°C] $x_4$	P [Pa] $x_5$	...
32.9	24.6	42.5	10.5	32.1	...
23.6	61.3	15.1	12.1	30.9	...

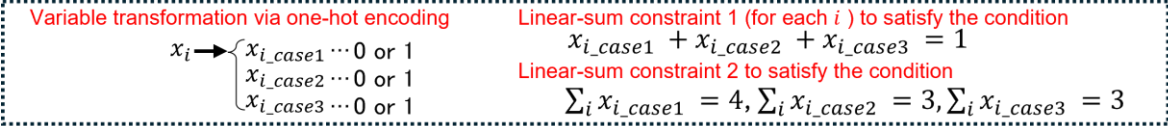
$x_1$  [mm]  
 $x_2$  [mm]

$x_1$  [mm]  
 $x_2$  [mm]

3. Constraints on Counts

Another practical use is setting count limits when the explanatory variables are categories. Imagine each item must be assigned to one of three categories (Case1, Case2, Case3). If you want exactly four items in Case1, three in Case2, and three in Case3, you can instruct Multi-Sigma® to keep those totals while searching for the best solution. In practice, define a single category field with three options (Case1, Case2, Case3). Each item selects exactly one option; one item, one case. Then you specify the desired totals, for example, Case1 = 4, Case2 = 3, Case3 = 3. Multi-Sigma® will run the optimization while respecting the counts.

$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	$x_6$	$x_7$	$x_8$	$x_9$	$x_{10}$
Case1	Case2	Case3	Case1	Case2	Case3	Case1	Case2	Case3	Case1
Case1	Case1	Case2	Case2	Case3	Case3	Case1	Case1	Case3	Case4
Case3	Case1	Case2	Case2	Case3	Case1	Case2	Case1	Case3	Case1
:	:	:	:	:	:	:	:	:	:



4. Constraints on Products

Another application is to constrain the product of explanatory variables. Use this with care: if the variables that make up the product already have other constraints, this method may not apply. For example, suppose the width and height of a rectangular design can vary, but the area must stay at or above a set value. By applying a logarithmic (log) transform, you can enforce the area requirement during optimization. In Multi-Sigma®, define variables such as  $\log(x_1)$  (and similarly for others).

